This technical annex provides background to the comparison between the Golden Rule and the Debt-reduction Rule, discussed in European Policy Brief 6. It provides both for intuitive and mathematical proof of figure 1 of that Policy Brief, which compares the stringency of the Golden Rule to that of the Debt-reduction Rule.

Intuitive Proof

The figure on page two of this technical annex illustrates the average yearly-required debt-reduction for both the Debt-reduction Rule and the Golden Rule. The black line represents the Debt-reduction Rule. The debt-reduction that is required by this Rule is independent of economic growth. This is different for the Golden Rule, where the debt-to-GDP reduction is also based on economic growth. The blue lines therefore indicate the decrease in the debt-to-GDP level that is required by the Golden Rule given different nominal growth levels (ranging from 1% to 4% nominal growth).

Given a specific nominal growth level, the Debt-reduction Rule becomes more stringent only if a country exceeds a specific debt-to-GDP threshold. This is shown in the figure on page two. For example, given a nominal growth level of 2%, the Debt-reduction Rule becomes more demanding only if a country has debt that exceeds 82.5% of GDP. If nominal growth is 3%, the tipping point is 120.5% of debt-to-GDP; in case of 4%, nominal growth the tipping point is a stunning 218.5% debt-to-GDP.
We can use these intersects to identify when the Debt-reduction Rule and the Golden Rule are equally restrictive. The curve in figure 1 of European Policy Brief 6 is composed of these intersects.

**Mathematical Proof**

In this mathematical proof, we provide an equation for which, given a growth and debt/GDP level, both the Debt-reduction Rule and the Golden Rule are equally strict. As the Golden Rule requires structural deficits of 0.5% of GDP or less, this implies that the Debt-reduction Rule should also impose a deficit limit of 0.5% of GDP at most.

We use the following symbols:
- $B = \text{government debt}$;
- $GDP = \text{Gross Domestic Product}$;
- $g = \text{nominal growth-to-GDP rate in year } 0$;
- $d = \text{government deficit in year } 0$;
- Subscripts indicate the relevant year. If no indication, the variable concerns year 0.

**Assumptions:**

$$B_0 + d = B_1$$

$$GDP_0 + GDP_0 \cdot g = GDP_1$$

The debt-reduction rule implies that:
We can use the assumptions to modify $B_1$ and $\text{GDP}_1$:

$$\frac{B_0}{\text{GDP}_0} - \left(\frac{B_0}{\text{GDP}_0}\right) - \frac{60\%}{20} = \frac{B_1}{\text{GDP}_1}$$

As we want to know in what case the Golden Rule and the Debt-reduction Rule are equally strict, we know that:

$$d = 0.5\% \times \text{GDP}_0$$

Therefore:

$$\frac{B_0}{\text{GDP}_0} - \left(\frac{B_0}{\text{GDP}_0}\right) - \frac{60\%}{20} = \frac{B_0 + (0.5\% \times \text{GDP}_0)}{\text{GDP}_0 + \text{GDP}_0 \times g}$$

We can isolate $g$ in a few steps:

$$\frac{B_0}{\text{GDP}_0} - \left[\frac{\left(\frac{B_0}{\text{GDP}_0}\right) - 60\%}{20}\right] = \frac{1}{\text{GDP}_0 + \text{GDP}_0 \times g}$$

We now reverse the numerators and the denominators:

$$\frac{B_0 + (0.5\% \times \text{GDP}_0)}{\text{GDP}_0} - \left[\frac{\left(\frac{B_0}{\text{GDP}_0}\right) - 60\%}{20}\right] = \text{GDP}_0 + \text{GDP}_0 \times g$$

We bring $\text{GDP}_0$ to the left-hand side of the equation:

$$\frac{B_0 + (0.5\% \times \text{GDP}_0)}{\text{GDP}_0} - \left[\frac{\left(\frac{B_0}{\text{GDP}_0}\right) - 60\%}{20}\right] - \text{GDP}_0 = \text{GDP}_0 \times g$$

This results in:

$$\frac{B_0 + (0.5\% \times \text{GDP}_0)}{\text{GDP}_0} - \left[\frac{\left(\frac{B_0}{\text{GDP}_0}\right) - 60\%}{20}\right] \div \text{GDP}_0 = g$$